Rateless Codes with Optimum Intermediate Performance

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Abstract—In this paper, we design several degree distributions for rateless codes with optimum intermediate packet recovery rates. In rateless coding, the employed degree distribution significantly affects the packet recovery rate. Each degree distribution is designed based on the number of message packets, $k$, and desired coding overhead, $\gamma$, which is the ratio of the number of received packets, $n$, to $k$, i.e., $\gamma = \frac{n}{k}$.

Previously designed degree distributions are tuned for full recovery of the entire source packets for $\gamma$’s slightly larger than 1, and as a consequence, they show very small packet recovery rates for $\gamma < 1$. Hence, finding degree distributions with maximal packet recovery rates in intermediate range, $0 < \gamma < 1$, is of interest. We define packet recovery rates at three values of $\gamma$ as our conflicting objective functions and employ NSGA-II multi-objective genetic algorithms optimization method to find several degree distributions with optimum packet recovery rates. We propose degree distributions for both cases of finite and infinite (asymptotic) $k$.

I. INTRODUCTION

Intermediate recovery rate is important in applications where partial recovery of the source packets from received encoded packets is still beneficial. For instance, in video or voice transmission, the receiver can benefit from incomplete recovered data by playing a lower quality of the media. This motivates the design of forward error correction (FEC) codes with high intermediate performance. Here, the term performance refers to the the ability of the employed coding scheme to recover the maximum number of source packets at the receiver.

Rateless codes are modern flexible FEC codes with low coding/decoding complexity. Since rateless codes do not impose a fixed coding rate, they can be employed on channels with varying or unknown loss rates. Previously designed rateless codes [1–3] offer efficient FEC codes for recovery of the entire message with low error probability while they show a weak performance in intermediate range. Therefore, it is of interest to design rateless codes with optimum intermediate performance.

The parameter that determines the packet recovery rate of rateless codes is the employed coding degree distribution. In rateless encoding, first a packet degree, $d$, is chosen from a degree distribution, $\{\Omega_1, \Omega_2, \ldots, \Omega_n\}$, where $\Omega_i$ is the probability that $d = i$ and $\sum_{i=1}^{n} \Omega_i = 1$. This degree distribution is also shown by its generator polynomial $\Omega(x) = \sum_{i=1}^{n} \Omega_i x^i$.

Next $d$ source packets are chosen uniformly at random from the source packets, and are XORed together to generate an encoded packet. This procedure is repeated until enough number of packets is collected at the receiver. In this paper, we denote the number of source packets by $k$, number of received coded packets by $n$, and received overhead by $\gamma$, where $\gamma = \frac{n}{k}$. Furthermore, we denote the ratio of number of recovered packets at the receiver to $k$ by $z$.

The decoding of rateless codes is performed in an iterative fashion. First, the decoder finds received coded packets of degree-one and recovers one source packet from each one of these coded packets. Next, by removing the recovered source packets from higher degree encoded packets more degree-one coded packets emerge. This iterative decoding continues until no more coded packets can be reduced to degree one.

The degree distribution of rateless codes are usually finely tuned to get the optimum results at fixed $\gamma$’s slightly larger than one, while here we prefer to have codes with superior performance in all intermediate range, $0 < \gamma < 1$. Note that, when the packet recovery rate at a certain $\gamma$ is increased by modifying the degree distribution, the recovery rate at other $\gamma$’s degrades. In other words, we have to deal with multiple dependant recovery rates. Consequently, if we consider recovery rates as conflicting objective functions, we have a multi-objective optimization problem.

Later we show that based on previous studies [4] the overhead range of $0 < \gamma < 1$ is divided into three regions. Therefore, we choose one fixed $\gamma$ from each region, i.e. $\gamma \in \{0.5, 0.75, 1\}$, and employ a multi-objective optimization method to find the degree distribution with maximal packet recovery rates at these three $\gamma$’s. Each $\gamma$ represents one region of intermediate range and the resulting degree distributions would perform optimally throughout intermediate range. Since we have three objective functions, the optimum answers will be a member of a 3D pareto front [5] in our objective space. In this 3D space, each dimension represents an objective function. Our decision space includes several continuous coefficients, $\Omega_i$, of each degree. Since our decision space is huge, we propose to employ the state of the art multi-objective genetic algorithm NSGA-II [5].

When the pareto front members are found, we define a total cost which is the weighted summation of difference of
packet recovery rates from the upper bound on recovery at each respective $\gamma$. In this way, we can choose one degree distribution among all optimum degree distributions according to the assigned weights.

We note that existing degree distributions are mostly optimized for asymptotic cases, i.e., when $k = \infty$. This will result in sub-optimality of designed codes when $k$ is finite. In this paper, first we take a similar approach to the existing studies and optimize the degree distributions in asymptotic case, and then we take a step further and consider finite message lengths. We show that the optimum degree distributions of rateless codes designed for finite message lengths have significant difference with asymptotic case.

The paper is organized as follows. Section II provides a background on existing rateless codes and studies their intermediate performance. In Section III, we define the problem and briefly describe some basic concepts of multi-objective genetic algorithms. Next, we design the optimum rateless codes using a multi-objective genetic algorithm for finite and infinite message lengths. We propose a weighted cost function in Section IV, and propose some degree distributions for special weights. Thereafter, we evaluate the performance of the proposed degree distributions. Finally, Section V concludes the paper.

II. RELATED WORK

Sanghavi in [4] has theoretically studied the intermediate performance of rateless codes. In [4] it is shown that $0 < \gamma < 1$ has three regions, and the upper bound on all degree distributions in each region is found. In the first region, $z \in [0, \frac{1}{2})$, we have $\gamma = -\log(1 - z)$, and the optimum degree distribution has degree one packets only, i.e., $\Omega_1 = 1, \Omega_{i \neq 1} = 0$. In other words, in this region uncoded packets have the highest intermediate recovery rate at the decoder.

For the second region, $z \in [\frac{1}{2}, \frac{3}{4})$, we have $\gamma = -\log(1 - z)$, and the optimum degree distribution has degree two packets only, i.e., $\Omega_2 = 1, \Omega_{i \neq 2} = 0$. Note that in practice the decoder needs packets of degree one to start decoding while we can see that this optimum degree distribution does not include packets of degree one. Since the optimum degree distributions of [4] are derived theoretically with asymptotic assumptions, they do not consider the real rateless decoding requirements. As a result, the optimum degree distributions might not perform well in practice with messages of finite length.

The third region of intermediate range is $z \in [\frac{3}{4}, 1]$. In this region, for an integer $m$, where $\frac{m-1}{m} < z < \frac{m}{m+1}$, we have $\gamma = \frac{m-1}{m} + \frac{1}{m^2} \sum_{i > m} \frac{z^i}{i}$. It is shown [4] that the optimum degree distribution in this region is given by

$$
\Omega_i = \begin{cases} 
\frac{1}{a(i-1)} & 2 < i < m - 1, \\
1 - \frac{m-2}{a(m-1)} & i = m \\
0 & o.w.
\end{cases} 
$$

In this way, we can choose one degree distribution among all optimum degree distributions according to the assigned weights.

Authors in [6] have proposed growth codes, which are originally designed for wireless sensor networks to maximize the data persistence by increasing the intermediate packet recovery rate. In growth coding, the encoder gradually increases the degree of encoded packets on the fly according to $z$ such that each delivered packet has the highest probability of decoding a source packet at the receiver. In other words, this coding tries to adapt the degree of coded packets such that the instantaneous decoding probability of each delivered packet is maximized. The drawback of growth codes is the assumption that encoder is aware of $z$. This assumption requires a several feedback from receiver, which may not be very practical. Interested readers are encouraged to refer to [6].

Luby in [3] proposed LT-codes, which are the first practical realization of rateless codes. LT codes degree distribution is called Robust-Soliton degree distribution. Robust-Soliton degree distribution is defined as a function of $k$ and two tunable parameters $c$ and $\delta$ [3]. Robust-Soliton degree distribution is the practical variation of its former theoretically derived degree distribution, ideal Soliton distribution.

Authors in [1, 2] have proposed two-layer rateless coding schemes which provide superior performance for full data recovery. In these coding schemes, packets are precoded with a high-rate conventional FEC code, and then, they are encoded employing a specific rateless code. Some degree distributions are proposed in [2] for different values of $\gamma$ and $k$.

The intermediate performance of rateless codes described in this section is illustrated in Figure 1. Note that in this figure we have implemented raptor codes without precoding. Precoding is important to achieve close to zero error floors, which is not a concern in intermediate packet recovery due to very large error rates.

Fig. 1. The packet recovery rates for raptor codes ($k = 65536$), growth codes, random packet forwarding, no coding with ideal channel, and the upper bound on all recovery rates [4].

In Figure 1, we have also depicted the case where packets are selected randomly from source packets and forwarded without coding. In contrast to what one might believe, random packet forwarding has superior intermediate performance at small overheads, $z \leq \frac{3}{4}$.
III. Rateless Codes Design

We are interested in finding degree distributions with maximal intermediate packet recovery rates. Since in $0 < \gamma < 1$ error rates are high, we do not need two layer codes similar to [1, 2]. Therefore, we try to design single layer rateless codes similar to LT codes. Furthermore, existing degree distributions are optimized for a fixed $\gamma$, while we expect a high recovery rate throughout the intermediate range. As a result, we propose a novel approach that finds degree distributions for high recovery rates throughout intermediate range.

As discussed earlier, intermediate range is divided into three regions for $z$ or equivalently for $\gamma$. We select one $\gamma$ from each region, i.e. $\gamma \in \{0.5, 0.75, 1\}$, and define three objective functions to be the packet recovery rates at these $\gamma$’s.

Codes designed for intermediate recovery have small-degree packets. For instance, in growth codes at $i > 50$ packet. For instance, in growth codes at $i > 50$, $\gamma = 0$. According to [4] in the second region with $z \in [\frac{1}{2}, \frac{2}{3}]$ only $\Omega_2$ is non-zero, and in the third region coefficients of $\Omega_3$ and higher degrees are zero for $z = 0.98$. Besides, these degree distributions are derived asymptotically, while the degree distributions have much smaller degrees in practice when $k$ is finite. The rationale behind having small degrees is that error rates are large in intermediate range, thus large-degree packets have smaller chance to decode a source packet.

Based on these observations, and the three selected $\gamma$’s, we consider degree distributions with maximum degree of 50, i.e. $\Omega_i = 0$ for $i > 50$. Thus, we have fifty continuous decision variables which should be tuned to concurrently maximize recovery rates at three $\gamma$’s. Later we shall see that we end up with much smaller packet degrees.

We take two approaches toward this problem. First we consider the asymptotic case similar to existing studies. In this case, we formulate packet recovery rates using a technique called And-Or tree analysis [1, 7–9]. In the second case, we consider finite-length rateless codes with $k = 100$ and $k = 1000$ and show how degree distributions vary with $k$.

A. Objective Functions

In And-Or tree analysis technique [1, 7–9] the error rate of iterative decoding of rateless codes is probabilistically formulated for $k = \infty$. Consider a rateless code with parameters $\Omega(x)$ and $\gamma$. Let $y_{l-1}$ be the probability that a packet is not recovered after $l$ decoding iterations. We have [7–9]

$$y_l = \delta(1 - \beta(1 - y_{l-1})), l \geq 1$$

(2) for $\gamma = 0.5, \gamma = 0.75$, and $\gamma = 1$, respectively. These objective functions show the packet error rate of each degree distribution according to the received overhead, $\gamma$. These are our conflicting objective functions. Improving one objective function will worsen the other objective functions.

In the second case, we assume that $k$ is limited to a fixed number. To the best of our knowledge, the packet error rate of rateless codes with finite $k$ is not formulated in a closed form yet. Thus we need another method to evaluate the packet error rate or equivalently packet recovery rate at rateless decoding with finite $k$. We propose to average over multiple Monte Carlo real decoding results to evaluate each degree distribution recovery rate. We consider two cases with message lengths of $k = 100$, and $k = 1000$. We average packet error rates of $10^5$ Monte Carlo real decoding rounds for $k = 100$, and $10^4$ rounds for $k = 1000$. We have similar objective functions $F_{\gamma=0.5}(\Omega(x))$, $F_{\gamma=0.75}(\Omega(x))$, and $F_{\gamma=1}(\Omega(x))$, which are found by real decoding in this case.

B. Multi-Objective Optimization Method

Since we are dealing with conflicting objective functions, we need to employ pareto optimality concept instead of simple maximum or minimum preference. Let $X$ and $\bar{x}$ denote the decision space and a decision vector, respectively. Let $F_1(\bar{x}), F_2(\bar{x}), \ldots, F_n(\bar{x})$ denote the conflicting objective functions. The problem is to find decision vectors that minimize/maximize all objective functions. In the simple case with single objective function, the problem boils down to conventional minimization/maximization problems. However, in multi-objective function problems, we have to deal with multiple optimum answers called pareto optimal.

First we need to introduce some definitions. For a minimization problem, $\bar{x}_1 \in X$ is said to be dominated by $\bar{x}_2 \in X$, or $\bar{x}_1 \prec \bar{x}_2$, if $\forall i \in \{1, \ldots, n\}$, $F_i(\bar{x}_1) \geq F_i(\bar{x}_2)$ and for at least one $i$, $F_i(\bar{x}_1) > F_i(\bar{x}_2)$. A non-dominated pareto front vector, $\bar{x}^*$, is a decision vector which no other decision vector can dominate it. In other words, in a minimization problem no other decision vector exists such that it would decrease some objective functions without deteriorating at least one other objective function compared to $\bar{x}^*$.

The set of all dominant solution vectors form pareto optimal set. The plot of objective functions of pareto optimal members in the objective space builds the pareto front. In Figure 2, the concept of pareto optimal and pareto front for a simple problem with two-objective functions and two decision variables is illustrated.

Figure 2 shows that no member can dominate pareto front members, and pareto front members do not dominate each other.

In rateless codes design, we define a problem with three objective functions, $F_{\gamma=0.5}(\Omega(x))$, $F_{\gamma=0.75}(\Omega(x))$, and $F_{\gamma=1}(\Omega(x))$, with fifty decision variables $\bar{x} = \{\Omega_1, \Omega_2, \ldots, \Omega_{50}\}$. We find the pareto front in the objective space employing NSGA-II [5] multi-objective optimization (MOP) genetic algorithm, and report the pareto optimal vectors, which show optimum degree distributions.
The results of our simulations are three 3D pareto fronts for asymptotic case and finite-lengths of $k = 100$ and $k = 1000$. The results for asymptotic case are depicted in Figure 3 in objective space.

The results of our simulations are three 3D pareto fronts for asymptotic case and finite-lengths of $k = 100$ and $k = 1000$. The results for asymptotic case are depicted in Figure 3 in objective space.

![Figure 2. Concept of pareto optimality, pareto front, and domination for a two-objective minimization problem with two decision variables, $x_1$ and $x_2$.](image1)

![Figure 3. The resulting 3D pareto front of optimization problem for asymptotic case. Each axis represents an objective function.](image2)

Note that each point in Figure 3 represents a degree distribution, and none of these degree distributions dominate another member in the shown pareto front. One should choose an appropriate degree distributions that best fits his desired recovery rate. The results cannot be reported in this paper due to huge number of members. Therefore we have made them available online at [10].

In the next section, we assign weights to each objective function in order to choose some degree distributions of special weights out of many available pareto optimal degree distributions.

**IV. Evaluation of Results**

In order to select some degree distributions in the pareto front, we define a weighted summing function, $F(\Omega(x))$, which evaluates the difference of the resulting packet recovery rates of each degree distribution at three values of $\gamma$ with the upper bound on recovery rate of all rateless codes [4] as shown in Figure 1. The upper bound on rateless codes recovery rates at $\gamma = 0.5$, $\gamma = 0.75$, and $\gamma = 1$ are $0.393469$, $0.5828$ and $1$, respectively. Therefore, we define $F(\Omega(x))$ as

$$F(\Omega(x)) = W_{\gamma=0.5}(0.393469 - (1 - F_{\gamma=0.5}(\Omega(x)))) + W_{\gamma=0.75}(0.5828 - (1 - F_{\gamma=0.75}(\Omega(x)))) + W_{\gamma=1}(1 - (1 - F_{\gamma=1}(\Omega(x))))$$

where $W_{\gamma=0.5}$, $W_{\gamma=0.75}$, and $W_{\gamma=1}$ are the weights assigned to each objective function.

**A. Evaluation of Asymptotic Case Results**

Using (3), the multi-objective problem boils down to a simple minimization problem. For each set of weights, we evaluate (3) for all members of the pareto front and choose the degree distribution that minimizes this function. Table I shows the optimum degree distributions found for some important weights for asymptotic case.

<table>
<thead>
<tr>
<th>Weights</th>
<th>Optimum degree distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \quad 1 \quad 1$</td>
<td>$\Omega(x) = 0.26599x + 0.70401x^2$</td>
</tr>
<tr>
<td>$1 \quad 0 \quad 0$</td>
<td>$\Omega(x) = x$</td>
</tr>
<tr>
<td>$0 \quad 1 \quad 0$</td>
<td>$\Omega(x) = 0.00003x + 0.99997x^2$</td>
</tr>
<tr>
<td>$0 \quad 0 \quad 1$</td>
<td>$\Omega(x) = 0.00005x + 0.99995x^2$</td>
</tr>
<tr>
<td>$0 \quad 0 \quad 1$</td>
<td>$\Omega(x) = 0.00003x + 0.99997x^2$</td>
</tr>
<tr>
<td>$4 \quad 1 \quad 1$</td>
<td>$\Omega(x) = x$</td>
</tr>
<tr>
<td>$1 \quad 4 \quad 1$</td>
<td>$\Omega(x) = 0.12409x + 0.8751x^2$</td>
</tr>
<tr>
<td>$1 \quad 1 \quad 4$</td>
<td>$\Omega(x) = 0.11900x + 0.24922x^2 + 0.3414x^3$</td>
</tr>
<tr>
<td>$0 \quad 1 \quad 4$</td>
<td>$\Omega(x) = 0.11900x + 0.24922x^2 + 0.3414x^3$</td>
</tr>
</tbody>
</table>

According to [4] the optimum degree distributions for regions $z \in [0, \frac{1}{4}]$ (equivalently $\gamma \in [0, 0.693]$) and $z \in [\frac{1}{4}, \frac{1}{2}]$ (equivalently $\gamma \in [0.693, 0.824]$) are all degree one and all degree two packets, respectively. This is in accordance with our optimized degree distributions in Table I. With the weights of $W_{\gamma=0.5} = 1$, $W_{\gamma=0.75} = 0$, and $W_{\gamma=1} = 0$, we have been looking for the degree distributions with optimum performance at $\gamma = 0.5$ only. As reported in Table I, the resulting degree distribution has degree one only. When we set weights to $W_{\gamma=0.5} = 0$, $W_{\gamma=0.75} = 1$, and $W_{\gamma=1} = 0$, the optimum reported degree distribution has a large degree-2 coefficient and a small degree-1 coefficient.

The degree distribution of the third region has different higher degrees as discussed in [4], and the degree distribution reported with the weights $W_{\gamma=0.5} = 0$, $W_{\gamma=0.75} = 0$, and $W_{\gamma=1} = 1$ is an optimum degree distribution in this region.

It is interesting to note that the proposed degree distribution for growth rates at $\gamma = 1$ is one of the members of our
pareto front. One can find any specific degree distribution by setting the desired weights and choosing one out of many available degree distributions [10]. Furthermore, other linear or non-linear functions or other selection methods can be deployed according to application to choose an appropriate degree distribution.

We compare the performance of the rateless codes employing proposed degree distributions for asymptotic case with the upper bound on rateless codes intermediate performance in Figure 4(a) and Figure 4(b).

![Graph](image)

(a) Performance of degree distribution of weights (1,0,0), (0,1,0), (0,0,1), and (1,1,1).

![Graph](image)

(b) Performance of degree distribution of weights (4,1,1), (1,4,1), (1,1,4), and (0,1,4).

Fig. 4. Comparison of the performance of the rateless codes employing designed degree distributions for asymptotic case with the upper bound on rateless codes intermediate performance.

In Figures 4(a) and 4(b), we can see that the designed degree distributions show a high performance and perform close to upper bound. These degree distributions are optimum in intermediate performance. As can be seen, according to the selected weights the resulting codes have the highest recovery rate at the $\gamma$ with the highest weight.

### B. Evaluation of Finite-Length Case Results

As we discussed earlier, the number of message packets is limited in practice and asymptotically optimized degree distributions result in suboptimal decoding. The optimized degree distributions for finite-length rateless coding of $k = 100$ and $k = 1000$ are reported in Tables II and III for some special set of weights [10].

#### TABLE II

<table>
<thead>
<tr>
<th>Weights</th>
<th>Optimum degree distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 1</td>
<td>$\Omega(x) = 0.348x + 0.622x^2$</td>
</tr>
<tr>
<td>1 0 0</td>
<td>$\Omega(x) = x$</td>
</tr>
<tr>
<td>0 1 0</td>
<td>$\Omega(x) = 0.191x + 0.808x^2 + 0.0003x^3$</td>
</tr>
<tr>
<td>0 0 1</td>
<td>$\Omega(x) = 0.116x + 0.467x^2 + 0.417x^3$</td>
</tr>
<tr>
<td>4 1 1</td>
<td>$\Omega(x) = x$</td>
</tr>
<tr>
<td>1 4 1</td>
<td>$\Omega(x) = 0.346x + 0.652x^2$</td>
</tr>
<tr>
<td>1 1 4</td>
<td>$\Omega(x) = 0.151x + 0.7903x^2 + 0.0581x^3$</td>
</tr>
</tbody>
</table>

#### TABLE III

<table>
<thead>
<tr>
<th>Weights</th>
<th>Optimum degree distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 1 1</td>
<td>$\Omega(x) = 0.3131x + 0.6809x^2$</td>
</tr>
<tr>
<td>1 0 0</td>
<td>$\Omega(x) = x$</td>
</tr>
<tr>
<td>0 1 0</td>
<td>$\Omega(x) = 0.0139x + 0.9861x^2$</td>
</tr>
<tr>
<td>0 0 1</td>
<td>$\Omega(x) = 0.0624x + 0.5407x^2 + 0.2222x^3 + 0.1737x^4$</td>
</tr>
<tr>
<td>4 1 1</td>
<td>$\Omega(x) = x$</td>
</tr>
<tr>
<td>1 4 1</td>
<td>$\Omega(x) = 0.1448x + 0.8552x^2$</td>
</tr>
<tr>
<td>1 1 4</td>
<td>$\Omega(x) = 0.0624 + 0.9315x^2$</td>
</tr>
</tbody>
</table>

Table II shows that the degree distributions designed for finite-length rateless codes differ significantly from the degree distributions designed for asymptotic case. We can observe an increase in the number of degree one packets. It is also observed that large degree packets are not present in the degree distributions. The reason that the ratio of degree one packets has increased in intermediate range with finite $k$ is that the decoder requires more degree one packets in order to start decoding. On the other hand, low-degree packets have higher probability of decoding a source packet when $k$ is small, thus large-degree packets have disappeared from degree distributions.

The same result is also observed when Tables II and III are compared. When $k = 100$, decoder requires larger fraction of degree one packets and lower degree packets are preferred. We can also see that degree two packets constitute a high percentage of encoded packets compared to packets of other degrees. It shows that packets of degree two have the highest probability of generating a source packet in rateless decoding.
We compare the performance of the rateless codes employing proposed degree distributions optimized for finite-length with the upper bound on rateless codes intermediate performance in Figures 5 and 6.

![Graph](image1.png)

Fig. 5. Comparison of the performance of the rateless codes employing designed degree distributions for $k = 100$ with the upper bound on rateless codes intermediate performance.

![Graph](image2.png)

Fig. 6. Comparison of the performance of the rateless codes employing designed degree distributions for $k = 1000$ with the upper bound on rateless codes intermediate performance.

We can see that the designed degree distribution show superior performances and have close to upper bound recovery rate. Here we remind that since the upper bound is derived asymptotically, the difference of recovery rate with the upper bound becomes more significant as $k$ decreases.

V. CONCLUSION

In this paper, we studied the intermediate performance of rateless codes and proposed to employ multi-objective genetic algorithms to find several optimum degree distributions in intermediate range. We used the state-of-the-art optimization algorithm NSGA-II to find the set of optimum degree distributions which are called pareto optimal. We considered two cases of finite and infinite messages lengths. The objective functions employed in both cases are the packet recovery rates at different intermediate overheads.

After finding the pareto front in objective space, we defined a function which finds the weighted sum of recovery rates for each designed degree distribution. By setting various weights, we found different optimum degree distributions among pareto optimal members.

Simulation results compare the performance of proposed degree distributions with theoretical upper bound and show that proposed optimum degree distributions fulfill the expectations in different regions of intermediate packet delivery.

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