Surface Velocity Computation of Debris Flows by Vector Field Measurements

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Abstract. The surface velocity of natural debris flows is one of the main physical parameters for developing effective disaster prevention techniques. The authors already presented the result of velocity vector field measurement of debris flow by the spatio temporal derivative space method. In this paper, we propose a multi resolution spatio temporal derivative space method, and apply the method to some real debris flow video images.

1 Introduction

A great deal of damage has been caused by debris flows in the world. The surface velocity of natural debris flow is one of the main physical parameters to be known for developing effective disaster prevention techniques. Many remote sensing measurement methods such as spatial filtering, laser-Doppler method, etc. have been proposed so far (Ato (1963); Yeh and Cummins (1964); Itakura et al. (1981); Itakura et al. (1989); Honda et al. (1994)) and applied to the surface velocity measurement of debris flows (Itakura et al. (1985); Itakura and Suwa (1989); Itakura et al. (1994)). Though we can measure the mean velocity of a debris flow by these methods, we can not know the velocity vector field of the flow. The knowledge of the velocity vector field may provide some important information about some characteristics of a debris flow. The authors already presented the result of velocity vector field measurement of debris flow by the Spatio Temporal Derivative Space Method (STDSM) (Ando, 1986) in Inaba et al. (1997). In this paper, first we point out the shortcomings of the conventional STDSM, then we propose a multi resolution spatio temporal derivative space method, and finally apply the method to some real debris flow video images.

2 Spatio Temporal Derivative Space Method

In this section, we survey briefly the Spatio Temporal Derivative Space Method according to (Ando, 1986). Let \( f(x,y,t) \) be the luminance of a pixel of a moving image whose spatial coordinates are \((x,y)\) at time \(t\). In the following, we use the symbols \( f_x, f_y \) and \( f_t \) for the partial derivatives \( \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \) and \( \frac{\partial f}{\partial t}\) respectively. The space \((f_x, f_y, f_t)\) is called spatio temporal derivative space. Now, we consider a moving object whose velocity \( V \) is given by \((u,v)\), where \( u \) is the \( x \)-component of the velocity vector and \( v \) is the \( y \)-component of the velocity vector. If the object is observed from a coordinate system whose velocity is exactly equal to the velocity of the object, it looks as if it stays still. Then, from the definition of derivative we have the following relation:

\[
\begin{align*}
\frac{Df}{Dt}|_{V=(u,v)} & = \frac{f(x+udt, y+vdrt, t+dt) - f(x, y, t)}{dt} \\
& = uf_x + vf_y + f_t = 0.
\end{align*}
\]

Let the product sum of derivatives be

\[
S_{ij} = \iint_{\Gamma} f_i(x, y, t) f_j(x, y, t) dx dy dt \quad (\text{for } i, j).
\]

Where, the region \( \Gamma \) is defined as the neighborhood of an observation point. And a matrix \( S \) defined by

\[
S = \begin{bmatrix}
S_{xx} & S_{xy} & S_{xt} \\ S_{yx} & S_{yy} & S_{yt} \\ S_{xt} & S_{yt} & S_{tt}
\end{bmatrix}.
\]

It can be proved that the determinant of \( S \) is equal to zero if and only if \( uf_x + vf_y + f_t = 0 \) holds in \( \Gamma \). In the real measurement, the determinant of \( S \) is not exactly zero for the turbulence of noise. To solve this problem we introduce a parameter \( J \) defined as:

\[
J = \iint_{\Gamma} (uf_x + vf_y + f_t)^2 dx dy dt.
\]
The velocity vector \((u, v)\) is chosen that minimizes the parameter \(J\). In order to calculate \((u, v)\), we assume that each of the derivatives of \(J\) for \(u\) and \(v\) are equal to zero. Then we obtain
\[
\begin{align*}
us_{xx} + vs_{xy} + st_{xt} &= 0, \\
us_{xy} + vs_{yy} + st_{yt} &= 0.
\end{align*}
\]
From these equations, the velocity vector \((u, v)\) is then obtained as follows:
\[
(u, v) = \left( \frac{S_{yy}S_{tt} - S_{xt}S_{xy}}{S_{xx}S_{yy} - S_{xy}^2}, \frac{S_{xt}S_{xy} - S_{yt}S_{xx}}{S_{xx}S_{yy} - S_{xy}^2} \right). \tag{5}
\]

### 3 Multi Resolution STDSM

#### 3.1 Error Index of STDSM

In 2, the accuracy of a resultant velocity vector by Eq.5 becomes rather low, if the denominator of Eq.5:
\[
J_{\text{det}} = S_{xx}S_{yy} - S_{xy}^2, \tag{6}
\]
is considerably small. Furthermore, if the distribution of \((f_x, f_y, f_z)\) is hardly approximated by a plane, the index \(J\) will become large. In Ando (1986), the error index is given by the following equation:
\[
E = \frac{(S_{xx} + S_{yy})J_{\text{min}}}{J_{\text{det}}|\Gamma|}, \tag{7}
\]
where, \(J_{\text{min}}\) is the minimized value of \(J\), and \(|\Gamma|\) is the area of \(\Gamma\), that is, the number of pixels included in \(\Gamma\). In this paper, the error index is a little modified as follows:
\[
E = \frac{(S_{xx} + S_{yy})J_{\text{min}}}{J_{\text{det}}}. \tag{8}
\]

#### 3.2 Relationship between Resolution and Error

In the measurement of random flow by STDSM, the choice of the measurement region \(\Gamma\) is very important. In case the flow is almost uniform, the error of measured velocity is smaller when the area of \(\Gamma\) is larger. Note that the spatial resolution at velocity measurement is worse when the area of \(\Gamma\) is larger. Namely, the measurement error and the spatial resolution make a trade-off relation. However, the flow of debris flow is not uniform generally. In this case, there may exist an optimal size of \(\Gamma\) for the surface condition of a debris flow. In order to confirm this, we apply STDSM to an artificial random moving image (Yoshida et al., 1982) that models a debris flow (Itakura et al., 1991). The generated artificial random moving image consists of small blocks (16[dot]x16[dot]) that flow in different directions. If the measurement region \(\Gamma\) is properly selected, that is, the size of \(\Gamma\) is 16[dot]x16[dot], the estimated velocity vectors are also the proper ones. When the region \(\Gamma\) is set incorrectly, the estimated velocity vectors are wrong.

### 3.3 Algorithm of Multi Resolution STDSM

As we mentioned in the previous section, it is important to choose the correct size of \(\Gamma\) in the measurement of debris flow by STDSM. In this section, we propose a new algorithm to obtain a proper size of \(\Gamma\) without increasing a great deal of computation. In this algorithm the size of \(\Gamma\) is determined by the error index \(E\) defined by Eq. 8. Each steps of the algorithm is shown in the following.

**[MR-STDSM Algorithm]**

**Step 1:** Calculate the partial derivatives \(f_x(x, y, t), f_y(x, y, t), f_z(x, y, t)\) for stored image planes.

**Step 2:** Set level \(L\) be 1. Let
\[
S_{ij}^{(L)}(x, y) = f_i(x, y, t)f_j(x, y, t),
\]
where \(i, j\) is one of \(x, y, \text{ or } t\).

**Step 3:** Calculate \(V^{(L)}(x, y) = (u(x, y), v(x, y))\) by Eq. 5 from \(S_{ij}^{(L)}\) and also calculate \(E^{(L)}(x, y)\) by Eq. 8.

**Step 4:** Calculate
\[
S_{ij}^{(L+1)}(x) = \sum_{l,m \in \{0,1\}} S_{ij}^{(L)}(x + l, y + m)
\]
recursively from level \(L\)'s summation. The error index \(E^{(L+1)}(x, y)\) and velocity \(V^{(L+1)}(x, y)\) is also calculated.

**Step 5:** If the level \(L + 1\)'s error index satisfies the following condition:
\[
E^{(L+1)}(x, y) < \frac{1}{4} \sum_{l,m \in \{0,1\}} E^{(L)}(x + l, y + m)
\]
then, permutate the level \(L\)'s vector \(V^{(L)}(x + l, y + m)\) (\(l, m \in \{0, 1\}\)) by the level \(L + 1\)'s vector \(V^{(L+1)}(x, y)\).

**Step 6:** Let the level be \(L = L + 1\), and repeat step 4 and 5 till a predefined level.

In this algorithm, since the product sum of derivatives of each level is calculated from the previous level summation recursively in step 4, the computation time is not so increased in comparison with the conventional STDSM. This is shown by Fig.1. We confirm that the correct size of \(\Gamma\) is chosen by MR-STDSM and therefore the proper result is obtained by using the artificial random moving image referred in 3.2.
4 Results for Debris Flow Videos

The measurement of velocity vectors by the MR-STDSM has been applied to some debris flow video images. One is the video tape of two natural debris flows which occurred on May 1, 1995 (Fig. 2) and June 3, 1995 at the Nojiri river of Mt. Sakurajima Volcano, Kagoshima Prefecture, which is located in the southern part of Japan. And the other is the video tape of a debris flow which occurred on July 8, 1996 at the Moscardo torrent in Italy (Fig. 5) (Arattano and Marchi, 1998).

The velocity vector is estimated by MR-STDSM with two successive frames. The measured velocity vector fields for the video images are shown in Figures. 3 and 6. The magnitude of the velocity is indicated by the luminance of the flow in the figures (More black indicates a faster movement).

It is difficult to analyze the error for these results, because we cannot measure the velocity vector field of the debris flow directly. In place of analyzing error directly, we applied another method. First, we generated a reconstructed image from the first video frame and the estimated velocity vector field. Each pixel of the first frame was respectively moved according to the estimated velocity vector. And then, the reconstructed image was compared with the second video frame. The difference between the reconstructed image and the second frame was expected to be small if the estimated velocity vector field was an accurate one. The example of reconstructed image is shown in Fig. 4. As can be seen from the figure, the estimated error is small as expected.
5 Conclusions

Precise information about the surface velocity is essential for developing counter measures against natural turbulent flow hazards. Knowledge of the velocity vector field instead of simple velocity magnitude gives more effective information about its countermeasures. A new method for measuring the velocity vector field of random flow, based on STDSM was proposed in this paper. Typical results of velocity vector field of natural debris flows measured by this new method were presented. The performance of this method was confirmed by a computer simulation. However, we have not shown directly that the vector field corresponds to the true vector field of superficial velocities. Comparison between the result by our method and the result by other methods remains a goal for future work.

It is expected that this velocimetry system may contribute to a more effective disaster prevention, a topic that has gained more and more importance in recent years.

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References