The Way They Move:
Tracking Multiple Targets with Similar Appearance

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Outline

• Introduction

• Dynamics-based Multi-tracklet Association
  • Generalized Linear Assignment Problem
  • Tracklet Dynamics and Similarity Measure

• Dynamics-based Similarity Computation

• Experiments

• Conclusions
• Introduction

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Tracking Problem

CHALLENGE:
Distinguishing similar looking targets is hard.
Proposed Method

- Uses motion dynamics as a cue to distinguish targets with similar appearance
- I) Formulating the problem as a generalized linear assignment (GLA)
- II) Using efficient algorithms to estimate these similarity measures
  - ADMM
  - IHTLS

*Stitch similar tracklets to final trajectories*
A generalization of the ordinary assignment problem.

Simple cases: *Knapsack problem*
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  - Generalized Linear Assignment Problem
  - Tracklet Dynamics and Similarity Measure
- **Dynamics-based Similarity Computation**
- Experiments
- Conclusions
GLA formulation

\[ \max_X \sum_{i=1}^{N} \sum_{j=1}^{N} P_{ij} X_{ij} \]

\[ \text{st. } \sum_{i=1}^{N} X_{ij} \leq 1 ; \sum_{j=1}^{N} X_{ij} \leq 1 ; X_{ij} \in \{0, 1\} \]

\( P_{ij} \): Dynamics Similarity between tracks

\( X_{ij} \): i is the predecessor of j when \( X_{ij} = 1 \)

Solved by [18]

Higher Order Dynamics Models

- Tracklet with same motion dynamics can be explained by a single regressor. [11]

\[ y_k = a_1 y_{k-1} + a_2 y_{k-2} + a_3 y_{k-3} + \ldots + a_n y_{k-n} \]

n-order regressive model
Tracklet with same motion dynamics can be explained by a single regressor.

\[ y_k = a_1 y_{k-1} + a_2 y_{k-2} + a_3 y_{k-3} + \ldots + a_n y_{k-n} \]

\[ H^{(m)}_{\alpha} = \begin{bmatrix} y_1 & y_2 & y_3 & \ldots & y_m \\ y_2 & y_3 & y_2 & \ldots & y_{m+1} \\ y_3 & y_4 & y_5 & \ldots & y_{m+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix} \]
Tracklet with **same motion** dynamics can be explained by a single regressor.

\[ y_k = a_1 y_{k-1} + a_2 y_{k-2} + a_3 y_{k-3} + \ldots + a_n y_{k-n} \]

**Hankel matrix**

\[ H_{\alpha}^{(m)} = \begin{bmatrix} y_1 & y_2 & y_3 & \ldots & y_m \\ y_2 & y_3 & y_2 & \ldots & y_{m+1} \\ y_3 & y_4 & y_5 & \ldots & y_{m+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix} \]

**Constant off-diagonals**
Higher Order Dynamics Models

- Tracklet with same motion dynamics can be explained by a single regressor.

\[ y_k = a_1 y_{k-1} + a_2 y_{k-2} + a_3 y_{k-3} + \ldots + a_n y_{k-n} \]

\[ H_{\alpha}^{(m)} = \begin{bmatrix}
    y_1 & y_2 & y_3 & \ldots & y_m \\
    y_2 & y_3 & y_2 & \ldots & y_{m+1} \\
    y_3 & y_4 & y_5 & \ldots & y_{m+2} \\
    \vdots & \vdots & \vdots & \ddots & \vdots \\
\end{bmatrix} \quad n = \text{rank} \]
• Tracklet with same motion dynamics can be explained by a single regressor.

\[ y_k = a_1 y_{k-1} + a_2 y_{k-2} + a_3 y_{k-3} + \ldots + a_n y_{k-n} \]

The rank of \( H \) measures the complexity of the dynamics

\[ H^{(m)}_\alpha = \begin{bmatrix} y_1 & y_2 & y_3 & \ldots & y_m \\ y_2 & y_3 & y_2 & \ldots & y_{m+1} \\ y_3 & y_4 & y_5 & \ldots & y_{m+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{bmatrix} \]

\[ n = \text{rank} \]
Similarity Measure

\[ P_{ij} = \begin{cases} 
-\infty & \text{if } \alpha_i \text{ and } \alpha_j \text{ conflict} \\
\frac{\text{rank}(H_{\alpha_i}) + \text{rank}(H_{\alpha_j})}{\min_{\beta_i^j} \text{rank}(H_{\alpha_{ij}})} - 1 & \text{otherwise}
\end{cases} \]

\[ \alpha_{ij} = [\alpha_i \ \beta_i^j \ \alpha_j] \]
Similarity Measure


\[
P_{ij} = \begin{cases} 
-\infty & \text{if } \alpha_i \text{ and } \alpha_j \text{ conflict} \\
\frac{-\infty \cdot \text{rank}(H_{\alpha_i}) + \text{rank}(H_{\alpha_j})}{\min_{\beta_i} \text{rank}(H_{\alpha_{ij}})} - 1 & \text{otherwise}
\end{cases}
\]

\[
P_{ij} = \frac{5 + 5}{5} = 1
\]

\[
P_{ij} = \frac{5 + 5}{8} = 0.25
\]
Gaps/Occlusions

\( \alpha_i \) \( \alpha_j \)
Finding missing values of a matrix is **NP-Hard!**
\[ H_{\alpha_{ij}} = A_{\alpha_{ij}} + E_{\alpha_{ij}}, \text{ such that } A_{\alpha_{ij}}, E_{\alpha_{ij}} \in S_H \]

Low Rank

Noise

Hankel Structure

\[
\begin{bmatrix}
 y_1 & y_2 & y_3 & \ldots & y_m \\
 y_2 & y_3 & y_2 & \ldots & y_{m+1} \\
 y_3 & y_4 & y_5 & \ldots & y_{m+2} \\
 \vdots & \vdots & \vdots & \ddots & \vdots \\
\end{bmatrix}
\]
• $H_{\alpha_{ij}} = A_{\alpha_{ij}} + E_{\alpha_{ij}}$, such that $A_{\alpha_{ij}}, E_{\alpha_{ij}} \in S_H$
Rank Estimation with Hankel Structure, Missing Data

\[ H_{\alpha_{ij}} = A_{\alpha_{ij}} + E_{\alpha_{ij}}, \text{ such that } A_{\alpha_{ij}}, E_{\alpha_{ij}} \in S_H \]

\[ \Omega = H_{\omega}^{(n+1)} \text{ is introduced to recover the missing data} \]

\[ (A + E)x = b + f \]

Hadamard product

Hankel Total Least Squares (HTLS) algorithm [25]

The first modification: Introduction an "indicator" binary vector to flag missing data and allow its recovery while performing inpainting to stitch tracklets with gaps.

\[ \alpha_i \quad \text{Gap} \quad \alpha_j \]

\( \omega: \text{indicator} \)
Modifications of HTLS

• The first modification: Introduction an “indicator” binary vector to flag missing data and allow its recovery while performing inpainting to stitch tracklets with gaps.

• The second modification: Run the algorithm, iteratively, for increasing rank values to find the optimal rank.

\[ \omega: \text{indicator} \]
\[ \alpha_i \]
\[ \alpha_j \]
\[ \text{Gap} \]

\[ n = n+1 \]
\[ \mu_\eta \leq \eta_{\text{max}} \]

\[ \eta: \text{noise} \]
Solving the overdetermined system $Ax \approx b$, [A b] has Hankel structure, all the anti-diagonals in [A b] are subject to error.

The STLN solution will find a minimum $[E f]$ which also has Hankel structure, so that $(A + E)x = b + f$

$$[E f] = \begin{pmatrix}
\eta_1 & \eta_2 & \cdots & \eta_{l+1} \\
\eta_2 & \eta_3 & \cdots & \eta_{l+2} \\
\vdots & \vdots & \ddots & \vdots \\
\eta_m & \eta_{m+1} & \cdots & \eta_{l+m}
\end{pmatrix},$$
the vector $\eta$ is defined as

$$\eta = (\eta_1 \quad \eta_2 \quad \cdots \quad \eta_{l+m})^T.$$ 

Note that the perturbation $E$ depends only on

$$(2.4) \quad \alpha \equiv (\eta_1 \quad \eta_2 \quad \cdots \quad \eta_{l+m-1})^T$$

and the perturbation on the right hand side is

$$f = (\eta_{l+1} \quad \eta_{l+2} \quad \cdots \quad \eta_{l+m})^T.$$ 

Therefore, the vectors $\alpha$ and $f$ can be represented as $\alpha = P_0 \eta$ and $f = P_1 \eta$, where

$$P_0 = (I_{(l+m-1) \times (l+m-1)} \quad O_{(l+m-1) \times 1}) \in \mathbb{R}^{(l+m-1) \times (l+m)},$$

and

$$P_1 = (0_{m \times l} \quad I_{m \times m}) \in \mathbb{R}^{m \times (l+m)}.$$
Now, the structured residual \( \hat{r} \) for the perturbation \([E \ f]\) is defined as

\[
\hat{r} = \hat{r}(\eta, x) = b + f - (A + E)x.
\]

Accordingly, the problem is formulated as the following constrained minimization problem [2]

\[
\min_{\eta, x} \|D\eta\|_p,
\]

subject to \( \hat{r} = 0 \).

We solve it by using the penalty method [2]

\[
\min_{\eta, x} \left\| \begin{pmatrix} \omega \hat{r}(\eta, x) \\ D\eta \end{pmatrix} \right\|_p.
\]
Solved by linear approximation, Let $\Delta \eta$ and $\Delta x$ represent a small change in $\eta$ and $x$, respectively.

\[
\hat{r}(\eta + \Delta \eta, x + \Delta x) = b + P_1(\eta + \Delta \eta) - (A + E + \Delta E)(x + \Delta x)
\approx b + P_1 \eta - (A + E)x + P_1 \Delta \eta
\approx -(A + E)\Delta x - \Delta Ex.
\]

Introducing a matrix $Y$ that satisfies [17]

\[
Ex = Y\alpha = YP_0\eta,
\]

\[
\min_{\Delta x, \Delta \eta} \left\| \begin{pmatrix} \omega(YP_0 - P_1) & \omega(A + E) \\ D & 0 \end{pmatrix} \begin{pmatrix} \Delta \eta \\ \Delta x \end{pmatrix} + \begin{pmatrix} -\omega \hat{r} \\ D\eta \end{pmatrix} \right\|_p.
\]

Detail in HTLS

\[ M \equiv \begin{pmatrix} \omega (Y P_0 - P_1) & \omega (A + E) \\ D & 0 \end{pmatrix} \]

has **Toeplitz structure** as shown in the following example

**Example 2.1.** Suppose

\[ E = \begin{pmatrix} \eta_1 & \eta_2 & \eta_3 \\ \eta_2 & \eta_3 & \eta_4 \\ \eta_3 & \eta_4 & \eta_5 \\ \eta_4 & \eta_5 & \eta_6 \end{pmatrix}, \quad \text{and} \quad f = \begin{pmatrix} \eta_4 \\ \eta_5 \\ \eta_6 \end{pmatrix}. \]

Then

\[ D^2 = \text{diag}(1, 2, 3, 4, 3, 2, 1), \quad Y = \begin{pmatrix} x_1 & x_2 & x_3 & 0 & 0 & 0 \\ 0 & x_1 & x_2 & x_3 & 0 & 0 \\ 0 & 0 & x_1 & x_2 & x_3 & 0 \\ 0 & 0 & 0 & x_1 & x_2 & x_3 \end{pmatrix} \]
Algorithm H-STLN

**Input** – A vector $b$, a matrix $A$, with Hankel structure in $[A \ b]$, and tolerance $tol$. $E = 0$, $f = 0$.

**Output** – Hankel matrix $[E \ f]$, structured residual $\hat{r}$, and vector $x$ such that $(A + E)x = b + f - \hat{r}$. Upon convergence, $\hat{r} \approx 0$.

1. Compute $x$ from $\min ||Ax - b||_p$. Form $Y$ from $x$. Set $\hat{r} = b - Ax$.

2. repeat

   (a) minimize $\Delta x, \Delta \eta \left\| \begin{pmatrix} \omega (YP_0 - P_1) & \omega (A + E) \\ D & 0 \end{pmatrix} \begin{pmatrix} \Delta \eta \\ \Delta x \end{pmatrix} + \begin{pmatrix} -\omega \hat{r} \\ D \eta \end{pmatrix} \right\|_p$.

   (b) Set $x := x + \Delta x$, $\eta := \eta + \Delta \eta$.

   (c) Form $E$ and $f$ from $\eta$, form $Y$ from $x$, $\hat{r} = b + f - (A + E)x$.

until ($||\Delta x|| \leq tol$ and $||\Delta \eta|| \leq tol$)
Algorithm 1: HTLS with Missing Data

Input: $\alpha$ sequence of length $l$, $\omega$ sampling sequence of length $l$, desired rank $n$
Output: $\hat{\alpha}$ inpainted and cleaned sequence, $\eta$ noise/perturbation, $x$
AR coefficients
Form $[A|b]_{(l-n+1) \times (n+1)}$
Solve $\min \|Ax - b\|^2_2$ for $x$
Form $P_1$ and $WD$ from $\omega$
$\eta = 0$
while $\left\| \begin{pmatrix} \delta \eta \\ \delta x \end{pmatrix} \right\| > \theta$ do
    Form $XP_0$ from $x$ and form $[E|f]_{(l-n+1) \times (n+1)}$
    Compute $r = b + f - (A + E)x$
    Form $M = \begin{pmatrix} \pi(P_1 - XP_0) & -\pi(A + E) \\ WD & 0 \end{pmatrix}$
    Solve $\min \left\| M \begin{pmatrix} \delta \eta \\ \delta x \end{pmatrix} + \left( \begin{pmatrix} \pi r \\ WD\eta \end{pmatrix} \right) \right\|_2^2$ for $\delta \eta, \delta x$
    Update $x = x + \delta x, \eta = \eta + \delta \eta$

$W: \text{diag}(\omega)$

$D: \text{a diagonal matrix with the number of times each } \eta_i \text{ appears in the Hankel matrix } H_{\eta}^{(n+1)}$
Algorithm 2: Iterative Hankel Total Least Squares

Input: $\alpha$ sequence of length $l$, $\eta_{\text{max}}$ maximum average error, $\omega$ sampling sequence

Output: $\tilde{\alpha}$ inpainted and cleaned sequence, $n$ estimated rank for the sequence

$n = 0$, $\mu_\eta = \text{huge}$

Form $\Omega_{(l-n)\times(n+1)} = H^{(n+1)}_\omega$

while $\mu_\eta > \eta_{\text{max}}$ do

$n = n + 1$;

Solve HTLS problem, $\min ||\Omega \circ [E|f]||_F$ st.

$(A + E)x = b + f$

Form $[E|f]_{(l-n)\times(n+1)} = \mathcal{H}(\eta)$

Compute average error, $\mu_\eta = \frac{||\Omega \circ [E|f]||_F}{||\Omega||_1}$

end while

min $\|\Omega \circ [E|f]\|_F$

st. $(A + E)x = b + f$

$[A|b],[E|f] \in S_n$

min $\|\Omega \circ [E|f]\|_F$

st. $(A + E)x = b + f$

$[A|b],[E|f] \in S_n$
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Execution Times

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**KSP** [8], J. Berclaz, F. Fleuret, E. Turetken, and P. Fua. Multiple object tracking using k-shortest paths optimization. IEEE Trans. PAMI, 2011
Evaluation

- **Multiple Object Tracking Accuracy**
  \[ MOTA = 1 - \frac{\sum_t (fn_t + fp_t + mm_t)}{\sum_t gt_t} \]

- **Miss Match Ratio**
  \[ MMR = \frac{\sum_t fn_t}{\sum_t tt_t} \]

- **False Negative Ratio**
  \[ FNR = \frac{\sum_t fn_t}{\sum_t gt_t} \]

- **False Positive Ratio**
  \[ FPR = \frac{\sum_t fn_t}{\sum_t pt_t} \]
Experiments - with 20% outliers

MOTA

MMR

FNR

FPR
Experiments (2) - with 12% missing data
Video Demo
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• Dynamics/Motion is a strong cue for identify. We can use target dynamics to compare tracklets.
• Rank of the Hankel matrix measures motion complexity.
• ADMM and IHTLS can find the rank of the Hankel matrix, even with missing values.