Wavelet Transformation

- The wavelet transform describes a multi-resolution decomposition process in terms of expansion of an Image onto a set of wavelet basis functions.
  - Wavelet transform vs. Short-term Fourier transform
- Wavelet compression process
  - Take wavelet transform of a digitized image.
  - Allocate bits among the wavelet coefficients.
- Wavelet basis function: well-localized in both space and frequency
- Dilation equations:
  \[ \phi(x) \sim \sum_k c_k \phi(2x - k), \quad \int \phi(x) dx = 1 \]
- Two multi-resolution decomposition schemes: pyramid and subband.

Wavelet-Based Image Coding

- Embedded Zero-tree Wavelet coding (EZW)
  - by J. M. Shapiro (1993)
- Set Partitioning in Hierarchical Trees (SPIHT)
  - by A. Said and W. A. Pearlman (1996)
    - http://www.cipr.rpi.edu/research/SPIHT/
- Layered Zero Coding (LZC)
  - by D. Taubman and A. Zakhor
- Rate-Distortion optimized Embedded coding (RDE)
  - by J. Li and S.-M. Lei
- Embedded Block Coding with Optimized Truncation (EBCOT)
  - by D. Taubman
  - An important part of JPEG2000 Part 1
Short-term Fourier Transform (STFT)

- If we have a very non-stationary signal, we would like to know not only the frequency components but when in time the particular frequency components occurred.

- Short-term Fourier Transform
  - We break the time signal $f(t)$ into pieces of length $T$ and apply Fourier analysis to each pieces. Thus, we obtain an analysis that is a function of both time and frequency.
  - Boundary effect, (Problem #1)
  - To reduce the boundary effect, we window each piece before we take the FT. If the window shape is given by $g(t)$, the STFT is given by
    $$ F(\tau, T) = \frac{1}{T} \int_{-T/2}^{T/2} f(t) g \ast (t - \tau) e^{j\omega t} dt. $$
  - If the window function $g(t)$ is Gaussian, the STFT is called the Gabor transform.
  - The problem with the STFT is the fixed window size. The window size should contain at least one cycle of the low-frequency component, however, it can not accurately localize the high frequency spurt.
  - Uncertainty principle: If we wish to have finer resolution in time, we end up with a lower resolution in the frequency domain.
Wavelets

- A wavelet of the mother function \( \phi(t) \),

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \phi \left( \frac{t-b}{a} \right).
\]

- We scale a function \( f(t) \) by replacing \( t \) with \( t/a \) and translate a function to the right or left by an amount \( b \) by replacing \( t \) by \( t-b \) or \( t+b \).
- If we want the scaled function to have the norm as the original function, we need to multiply it by \( 1/\sqrt{a} \). Because

\[
\left\| f \left( \frac{\cdot}{a} \right) \right\| \leq \frac{1}{\sqrt{a}} \| f \|.
\]

- Wavelet coefficients of a function \( f(t) \),

\[
\omega_{a,b} = \frac{1}{\sqrt{a}} \int f(t) \psi_{a,b}(t) dt.
\]

- We can recover the function \( f(t) \) by

\[
f(t) = \frac{1}{c_y} \int \int \frac{dadb}{a^2} \omega_{a,b} \psi_{a,b}(t) \psi_{a,b}(t).
\]

where \( c_y = \frac{1}{\int \int \int d^2 \omega(\omega,\tau)} \)
Properties of Wavelet

- $C$, should be finite, so $\phi(0) = 0$. Since $\phi(0)$ is the average value of $\phi(t)$; therefore a requirement on the mother wavelet is that it has zero mean.
- We would also like the wavelets to have finite energy. Using Parseval's relationship, we can write this requirement as

$$\int_{-\infty}^{\infty} |\phi(t)|^2 \, dt = 1.$$  

For this to happen, $|\phi(t)|^2$ have to decay as $t$ goes to infinite. This requirements mean the energy in $\phi(t)$ is concentrated in a narrow frequency band.

- Discrete version: $a \cdot 2^m$, $b \cdot n2^m$.

$$\phi_{m,n} \cdot 2^{m/2} (2^m t \cdot n).$$

$$f(t) \cdot \sum_{m} \sum_{n} \phi_{m,n} \cdot w_{m,n} (t).$$

Multi-resolution Analysis and the Scaling Functions

- Approximated by scaling functions

$$f(t) \cdot \sum_{k} a_k \cdot \phi_k (t \cdot k).$$

- Harr scaling functions

$$\phi(t) \cdot \begin{cases} 1 & 0 \leq t \leq 1 \\ 0 & \text{otherwise}. \end{cases}$$

- Dilation version of the “mother” scaling function: In fact, we can obtain scaling functions at different resolutions in a manner similar to the procedure used for wavelets,

$$\phi_{j,k} (t) \cdot 2^{j/2} \cdot (2^j t \cdot k).$$

- In Harr

$$\phi_{1,0} (t) \cdot \begin{cases} \sqrt{2} & 0 \leq t \leq 1/2 \\ 0 & \text{otherwise}. \end{cases}$$
Wavelet Approximations by Harr (1)

\[ f(t) \]

\[ \psi^{(0)}_j(t) \]

\[ c_{0,k} \]

\[ f(t) \]

\[ \int c_{0,k} \psi^{(0)}_j(t) dt \]

(a)

Wavelet Approximations (2)
Dilation Equations

- The scaling function itself can be represented by its dilations at a high resolution:
  \[ \phi(t) \approx \sum_{k} h_k \phi_{1,k}(t). \]
- Multi-resolution analysis (MRA) equations:
  \[ \phi(t) \approx h_k \sqrt{2}\phi(2t - k). \]
  by substituting \( \phi_{1,k}(t) \approx \sqrt{2}\phi(2t - k). \)
- Harr scaling function:
  \[ \phi_n(t) \sim h_0 \phi_n - \frac{1}{\sqrt{2}} h_1 \phi_n - \frac{1}{\sqrt{2}} h_k \phi_{n-1} \quad \text{for } k \geq 1. \]
- Orthonormal transforms
  \[ \phi(t) = \int_{-\infty}^{\infty} \left( \sum_{m} \phi_m(t - m) \right) \, dt. \]

---

1-D Pyramidal Decomposition

\[
\begin{bmatrix}
 y_1 \\
 y_2 \\
 y_3 \\
 y_4 \\
 y_5 \\
 y_6 \\
 y_7 \\
 y_8 \\
 y_9 \\
 y_{10} \\
 y_{11} \\
 y_{12} \\
 y_{13} \\
 y_{14} \\
 y_{15} \\
 y_{16}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 s_1 \\
 d_1 \\
 s_2 \\
 d_2 \\
 s_3 \\
 d_3 \\
 s_4 \\
 d_4 \\
 s_5 \\
 d_5 \\
 s_6 \\
 d_6 \\
 s_7 \\
 d_7 \\
 s_8 \\
 d_8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 s_1 \\
 D_1 \\
 s_2 \\
 D_2 \\
 s_3 \\
 D_3 \\
 s_4 \\
 D_4 \\
 s_5 \\
 D_5 \\
 s_6 \\
 D_6 \\
 s_7 \\
 D_7 \\
 s_8 \\
 D_8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 S_1 \\
 S_2 \\
 S_3 \\
 S_4 \\
 S_5 \\
 S_6 \\
 S_7 \\
 S_8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 D_1 \\
 D_2 \\
 D_3 \\
 D_4 \\
 D_5 \\
 D_6 \\
 D_7 \\
 D_8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 S_1 \\
 S_2 \\
 S_3 \\
 S_4 \\
 S_5 \\
 S_6 \\
 S_7 \\
 S_8
\end{bmatrix}
\rightarrow
\begin{bmatrix}
 D_1 \\
 D_2 \\
 D_3 \\
 D_4 \\
 D_5 \\
 D_6 \\
 D_7 \\
 D_8
\end{bmatrix}
\]
Daubechies Wavelet DAUB4

- 4 coefficients

\[
\begin{align*}
? & \ c_0^2 \ ? \ c_1^2 \ ? \ c_2^2 \ ? \ c_3^2 \ ? \ 1 \\
\ ? & \ c_2 \ c_0 \ ? \ c_1 \ c_3 \ ? \ 0 \\
? & \ c_3 \ ? \ c_2 \ ? \ c_1 \ ? \ c_0 \ ? \ 0 \\
\ ? & \ 0 \ c_3 \ ? \ 1 \ c_2 \ ? \ 2 \ c_1 \ ? \ 3 \ c_0 \ ? \ 0 \\
\end{align*}
\]

\[
c_0 \ ? \ (1 \ ? \ \sqrt{3} ) / 4 \ \sqrt{2} , c_2 \ ? \ (3 \ ? \ \sqrt{3} ) / 4 \ \sqrt{2} \\
c_2 \ ? \ (3 \ ? \ \sqrt{3} ) / 4 \ \sqrt{2} , c_3 \ ? \ (1 \ ? \ \sqrt{3} ) / 4 \ \sqrt{2}
\]


Daubechies Wavelet DAUBx (2)

- Its derivative exists only almost everywhere (fails on points p/2^n)
- Left differentiable, but not right differential.
- The 130\textsuperscript{th} largest wavelet coefficient has an amplitude less than 10^{-5} of the largest coefficient among 1024 coefficients.
- Compact wavelets are better for lower accuracy approximation and for functions with discontinuities (such as edges), while small wavelets are better for achieving high numerical accuracy.

BAUB6, p=3 moment vanishing
\[
\begin{align*}
c_0 &= (1 + \sqrt{10} + \sqrt{5} + 2\sqrt{10})/16\sqrt{2} \\
c_2 &= (10 - 2\sqrt{10} + 2\sqrt{5} + 2\sqrt{10})/16\sqrt{2} \\
c_4 &= (5 + \sqrt{10} - 3\sqrt{5} + 2\sqrt{10})/16\sqrt{2} \\
c_1 &= (5 + \sqrt{10} + 3\sqrt{5} + 2\sqrt{10})/16\sqrt{2} \\
c_3 &= (10 - 2\sqrt{10} - 2\sqrt{5} + 2\sqrt{10})/16\sqrt{2} \\
c_5 &= (1 + \sqrt{10} - \sqrt{5} + 2\sqrt{10})/16\sqrt{2}
\end{align*}
\]
Embedded Zerotree Wavelet (1)

- **Embedded coding**
  - The bits are ordered in their importance
    - The encoding improves as more bits are transmitted.
    - The codec can terminate at any point thereby allowing a target rate or distortion metric to be met exactly.

- **Assumption**: most energy is compacted into lower bands
  - The coefficients closer to the root have higher magnitudes than coefficients further from the root.
EZW (2)

- Embedded Image Coding using Zerotrees of wavelet coefficients: DWT, prediction of the absence of significant information, entropy-coded successive-approximate quantization and lossless data compression.
- Zerotree structure
  - If a coefficient has a magnitude less than a given threshold, all its descendants might have magnitudes less than that threshold.
  - Significant coefficient at level $T$
    - If $|x| < T$
- Symbols
  - Zerotree root (ZR)
    - all descendants (including itself) are insignificant
  - Isolated zero (IZ)
    - the coefficient is insignificant but has some significant descendant
  - Significant positive (SP)
  - Significant negative (SN)

EZW (3)

- EZW is a multiple-pass algorithm
  - Each pass consists of 2 steps
    - Significance map encoding (dominant pass)
    - Refinement (subordinate pass)
- Two separate lists are maintained
  - Dominant list
    - coordinates of the coefficients that have not been found to be significant in the relative order
  - Subordinate list
    - magnitudes of the coefficients that have been found to be significant
- Successive Approximation
  - During a subordinate pass
    - the width of the effective quantizer step size (uncertainty interval) is cut in half
      - 1: the value falls in the upper half of the uncertainty interval
      - 0: the value falls in the lower half
    - The threshold is halved before each dominant pass
Flow Chart for Encoding a Coefficient

EZW (4)

- Example 1
  - Initial threshold
    \[ T_0 = \frac{2^{\log_2 c_{\max}}}{2} \]
  - \( c_{\max} \) will be in \([T_0, 2T_0]\)

<table>
<thead>
<tr>
<th>Subband</th>
<th>Coeff.</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>LL_2</td>
<td>26</td>
<td>SP</td>
</tr>
<tr>
<td>HL_2</td>
<td>6</td>
<td>ZR</td>
</tr>
<tr>
<td>LH_2</td>
<td>-7</td>
<td>ZR</td>
</tr>
<tr>
<td>HH_2</td>
<td>7</td>
<td>ZR</td>
</tr>
</tbody>
</table>

Dominant list
- "0": [16, 24]
- "1": [24, 32]

Subordinate list
- Coef Symbol Recon
  - 26 1 28
**EZW (5)**

- **Example 1 (cont.)**
  - Pass 1
    - \( T_1 = T_0 / 2 = 8 \)
    - \( T_2 = T_1 / 2 = 4 \)

<table>
<thead>
<tr>
<th>( X )</th>
<th>6</th>
<th>13</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>-7</td>
<td>7</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>-4</td>
<td>4</td>
<td>-3</td>
</tr>
<tr>
<td>2</td>
<td>-2</td>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

**Dominant list**

<table>
<thead>
<tr>
<th>Subband</th>
<th>Coef</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>HL</td>
<td>6</td>
<td>IZ</td>
</tr>
<tr>
<td>LH</td>
<td>-7</td>
<td>ZR</td>
</tr>
<tr>
<td>HH</td>
<td>7</td>
<td>ZR</td>
</tr>
<tr>
<td>HL</td>
<td>13</td>
<td>SP</td>
</tr>
<tr>
<td>HH</td>
<td>10</td>
<td>SP</td>
</tr>
<tr>
<td>HL</td>
<td>6</td>
<td>IZ</td>
</tr>
<tr>
<td>HL</td>
<td>4</td>
<td>IZ</td>
</tr>
</tbody>
</table>

**Subordinate list**

```
[8, 16) [16, 24) [24, 32)  
"0" : [8, 12) [16,20) [24,28)  
"1" : [12, 16) [20,24) [28,32)
```

- **Example 2**
  - Initial threshold
    - \( T_0 = 2^\log_2 c_{max} \) ? \( 2^\log_2 63 \) ? \( 2^5 \) ? 32

<table>
<thead>
<tr>
<th>Coef</th>
<th>Symbol</th>
<th>Recon</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>SP</td>
<td>56</td>
</tr>
<tr>
<td>-34</td>
<td>SN</td>
<td>40</td>
</tr>
<tr>
<td>-31</td>
<td>IZ</td>
<td>40</td>
</tr>
<tr>
<td>23</td>
<td>ZR</td>
<td>26</td>
</tr>
<tr>
<td>49</td>
<td>SP</td>
<td>26</td>
</tr>
<tr>
<td>10</td>
<td>ZR</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>ZR</td>
<td>13</td>
</tr>
<tr>
<td>-13</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>15</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>-7</td>
<td>ZR</td>
<td>40</td>
</tr>
<tr>
<td>14</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>-9</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>-7</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>13</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>ZR</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>-1</td>
<td>ZR</td>
<td>40</td>
</tr>
<tr>
<td>47</td>
<td>SP</td>
<td>56</td>
</tr>
<tr>
<td>-3</td>
<td>ZR</td>
<td>40</td>
</tr>
<tr>
<td>-2</td>
<td>ZR</td>
<td>40</td>
</tr>
</tbody>
</table>

Chiou-Ting Hsu, NTHU CS (CS531 Spring 2001) 21

**EZW (6)**

- **Example 2**
  - Initial threshold
    - \( T_0 = 2^\log_2 c_{max} \) ? \( 2^\log_2 63 \) ? \( 2^5 \) ? 32

<table>
<thead>
<tr>
<th>Coef</th>
<th>Symbol</th>
<th>Recon</th>
</tr>
</thead>
<tbody>
<tr>
<td>63</td>
<td>SP</td>
<td>56</td>
</tr>
<tr>
<td>-34</td>
<td>SN</td>
<td>40</td>
</tr>
<tr>
<td>-31</td>
<td>IZ</td>
<td>40</td>
</tr>
<tr>
<td>23</td>
<td>ZR</td>
<td>26</td>
</tr>
<tr>
<td>49</td>
<td>SP</td>
<td>26</td>
</tr>
<tr>
<td>10</td>
<td>ZR</td>
<td>13</td>
</tr>
<tr>
<td>14</td>
<td>ZR</td>
<td>13</td>
</tr>
<tr>
<td>-13</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>15</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>-7</td>
<td>ZR</td>
<td>40</td>
</tr>
<tr>
<td>14</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>-9</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>-7</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>4</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>13</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>3</td>
<td>ZR</td>
<td>40</td>
</tr>
<tr>
<td>4</td>
<td>ZR</td>
<td>56</td>
</tr>
<tr>
<td>-1</td>
<td>ZR</td>
<td>40</td>
</tr>
<tr>
<td>47</td>
<td>SP</td>
<td>56</td>
</tr>
<tr>
<td>-3</td>
<td>ZR</td>
<td>40</td>
</tr>
<tr>
<td>-2</td>
<td>ZR</td>
<td>40</td>
</tr>
</tbody>
</table>

Chiou-Ting Hsu, NTHU CS (CS531 Spring 2001) 22
**EZW (7)**

- Example 2 (2\textsuperscript{nd} pass):
  - Threshold $T_1 = 16$

<table>
<thead>
<tr>
<th>Subband</th>
<th>Coeff.</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LH_3$</td>
<td>-31</td>
<td>SN</td>
</tr>
<tr>
<td>$HH_3$</td>
<td>23</td>
<td>SP</td>
</tr>
<tr>
<td>$HL_2$</td>
<td>10</td>
<td>ZR</td>
</tr>
<tr>
<td>$HL_2$</td>
<td>-13</td>
<td>ZR</td>
</tr>
<tr>
<td>$LH_2$</td>
<td>14</td>
<td>ZR</td>
</tr>
<tr>
<td>$LH_2$</td>
<td>15</td>
<td>ZR</td>
</tr>
<tr>
<td>$LH_2$</td>
<td>14</td>
<td>ZR</td>
</tr>
<tr>
<td>$LH_2$</td>
<td>-9</td>
<td>ZR</td>
</tr>
<tr>
<td>$LH_2$</td>
<td>-7</td>
<td>ZR</td>
</tr>
<tr>
<td>$HH_2$</td>
<td>3</td>
<td>ZR</td>
</tr>
<tr>
<td>$HH_2$</td>
<td>-12</td>
<td>ZR</td>
</tr>
<tr>
<td>$HH_2$</td>
<td>-14</td>
<td>ZR</td>
</tr>
<tr>
<td>$HH_2$</td>
<td>8</td>
<td>ZR</td>
</tr>
</tbody>
</table>

Dominant list: $[16, 32, 48, 64)$

Subordinate list: $[0]: [16, 24, 32, 40, 48, 56)$

- $1$: $[24, 32, 40, 48, 56)$

**SPIHT (1)**

- Set Partitioning in Hierarchical Trees
  - Use a partitioning of the *spatial orientation trees* in a manner than tends to keep insignificant coefficients together in a larger subsets
  - Partitioning decision -> binary decision
    - Provide a significance map encoding that is more efficient than EZW
  - Each pass consists of 2 steps
    - Significance map encoding (set partitioning and ordering step)
    - Refinement

- Partitioning types
  - $O(i,j)$
    - Set of coordinates of the offsprings at $(i,j)$
  - $D(i,j)$
    - Set of all descendants of the coefficients at $(i,j)$
  - $H$
    - Set of all root nodes
  - $L(i,j)$
    - $L(i,j) = D(i,j) \cdot O(i,j)$
SPIHT (2)

- **Significant**
  - A set D(i,j) or L(i,j) is significant if any coefficient in the set has a magnitude greater than the threshold.

- **Three lists are maintained**
  - LIP (list of insignificant pixels)
    - Is initialized with the set H
  - LSP (list of significant pixels)
    - Is initially empty
  - LIS (list of insignificant sets)
    - Contain the coordinates of the roots of types D or L

- **At each pass**
  - Significant map encoding step
    - Process the members of LIP
      - If the coefficient is significant => transmit (1 + sign), move to LSP
      - Otherwise => transmit 0
    - Then process the members of LIS
      - Insignificant set => 0
  - Refinement step
    - Process the elements of LSP

---

SPIHT (3)

- **Example**
  - Initialize
    \[
    n \text{ ? } \log_2 c_{\text{max}} \text{ ? } \log_2 26 \text{ ? } 4
    \]
    \[
    T = 16
    \]
  - 1st pass
    - LIP \( \{ (0,0) \rightarrow 26, (0,1) \rightarrow 6, (1,0) \rightarrow -7, (1,1) \rightarrow 7 \} \)
    - LIS \( \{ (0,1)D, (1,0)D, (1,1)D \} \)
    - LSP \( \{ \} \)
      - LIP
        - 26 \( \rightarrow 1 + 0 \rightarrow \) move to LSP
          - 6 \( \rightarrow > 0 \)
          - -7 \( \rightarrow > 0 \)
          - 7 \( \rightarrow > 0 \)
        - LIS
          - (0,1)D \( \rightarrow > 0 \)
          - (1,0)D \( \rightarrow > 0 \)
          - (1,1)D \( \rightarrow > 0 \)
SPIHT(4)

• Example (cont.)
  – 2\textsuperscript{nd} pass
    • \( n \rightarrow n = 3, T = 8 \)
    • LIP \{ (0,1)\rightarrow 6, (1,0)\rightarrow -7, (1,1)\rightarrow 7 \}
    • LIS \{ (0,1)D, (1,0)D, (1,1)D \}
    • LSP \{ (0,0)\rightarrow 26 \}
      • LIP
        – 6 \rightarrow 0
        – -7 \rightarrow 0
        – 7 \rightarrow 0
      • LIS
        – (0,1)D \rightarrow 1
          » 13 \rightarrow 1+0 \Rightarrow \text{move to LSP}
          » 10 \rightarrow 1+0 \Rightarrow \text{move to LSP}
          » 6 \rightarrow 0 \Rightarrow \text{move to LIP}
          » 4 \rightarrow 0 \Rightarrow \text{move to LIP}
        – (1,0)D \rightarrow 0
        – (1,1)D \rightarrow 0
      • LSP
        – 26=(11010)_2 \Rightarrow 1

SPIHT (5)

• Example (cont.)
  – 3\textsuperscript{rd} pass
    • \( n \rightarrow n = 2, T = 4 \)
    • LIP \{ (0,1)\rightarrow 6, (1,0)\rightarrow -7, (1,1)\rightarrow 7, (1,2)\rightarrow 6, (1,3)\rightarrow 4 \}
    • LIS \{ (1,0)D, (1,1)D \}
    • LSP \{ (0,0)\rightarrow 26, (0,2)\rightarrow 13, (0,3)\rightarrow 10 \}
      • LIP
        – 6 \Rightarrow 1+0 \Rightarrow \text{move to LSP}
        – -7 \Rightarrow 1+1 \Rightarrow \text{move to LSP}
        – 7 \Rightarrow 1+0 \Rightarrow \text{move to LSP}
        – 6 \Rightarrow 1+0 \Rightarrow \text{move to LSP}
        – 4 \Rightarrow 1+0 \Rightarrow \text{move to LSP}
      • LIS
        – (0,1)D \Rightarrow 1
          » 4 \Rightarrow 1+0 \Rightarrow \text{move to LSP}
          » -4 \Rightarrow 1+1 \Rightarrow \text{move to LSP}
          » 2 \Rightarrow 0 \Rightarrow \text{move to LIP}
          » -2 \Rightarrow 0 \Rightarrow \text{move to LIP}
        – (1,1)D \Rightarrow 1
          » 4 \Rightarrow 1+0 \Rightarrow \text{move to LSP}
          » -3 \Rightarrow 0 \Rightarrow \text{move to LIP}
          » -2 \Rightarrow 0 \Rightarrow \text{move to LIP}
          » 0 \Rightarrow 0 \Rightarrow \text{move to LIP}
      • LSP
        – 26\Rightarrow 0, 13 \Rightarrow 1, 10\Rightarrow 0

\begin{tabular}{|c|c|c|c|}
\hline
26 & 6 & 13 & 10 \\
\hline
-7 & 7 & 6 & -4 \\
\hline
4 & -4 & 4 & -3 \\
\hline
2 & -2 & -2 & 0 \\
\hline
\end{tabular}